IOWA End-of-Course Assessment Programs

Released Items

ALGEBRA II

1 **Which cubic equation has roots of −2, 1, and 3?**

A $x^3 - 6 = 0$

INCORRECT: The student wrote a cubic equation with a constant that is the product of the roots.

B $x^3 + 6 = 0$

INCORRECT: The student wrote a cubic equation with a constant that is the absolute value of the product of the roots.

C $x^3 - 2x^2 - 5x + 6 = 0$

CORRECT:

$$(x + 2)(x - 1)(x - 3) = 0$$
$$(x^2 - x + 2x - 2)(x - 3) = 0$$
$$(x^2 + x - 2)(x - 3) = 0$$
$$x^3 - 3x^2 + x^2 - 3x - 2x + 6 = 0$$
$$x^3 - 2x^2 - 5x + 6 = 0$$

D $x^3 + 2x^2 - 5x - 6 = 0$

INCORRECT: The student incorrectly found the squared term and constant of the cubic equation.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities

2 **If $f(x) = 2x + 3$ and $g(x) = x - 2$, what is $f(g(3))$?**

A 1

INCORRECT: The student only evaluated $g(3)$.

B 5

CORRECT:

$$\begin{aligned} f(g(3)) &= f(3 - 2) \\ &= f(1) \\ &= 2(1) + 3 \\ &= 5 \end{aligned}$$

C 7

INCORRECT: The student evaluated $g(f(3))$.

D 10

INCORRECT: The student added the like terms of the functions together and then simplified:
$f(g(3)) \rightarrow (2x + 3) + (x - 2) = 3x + 1$
$3(3) + 1 = 10$

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions

3 **If $f(x) = 3x + 1$, what is $f^{-1}(x)$?**

A $\frac{1}{3}x - \frac{1}{3}$

CORRECT: $f(x) \rightarrow y = 3x + 1$

$f^{-1}(x) \rightarrow x = 3y + 1$

$x - 1 = 3y$

$\frac{x-1}{3} = y$

$\frac{1}{3}x - \frac{1}{3} = y$

B $\frac{1}{3}x - 1$

INCORRECT: The student did not divide both terms of the left-hand side of the equation by 3.

C $\frac{1}{3x} + 1$

INCORRECT: The student incorrectly found the reciprocal of $f(x)$:

$\frac{1}{f(x)} = \frac{1}{3x + 1} \rightarrow \frac{1}{3x} + 1$

D $-3x - 1$

INCORRECT: The student found the opposite of $f(x)$:

$-f(x) = -(3x + 1)$

$= -3x - 1$

CCSS Conceptual Category:
Functions

CCSS Domain:
Building Functions

4 **What is the solution to the following equation?**

$$3^{\frac{x}{5}} = 10$$

A $x = \dfrac{5}{\log 3}$

CORRECT:

$$3^{\frac{x}{5}} = 10$$

$$\frac{x}{5} \log 3 = \log 10$$

$$x \log 3 = 5 \log 10 \qquad (\log 10 = 1)$$

$$x = \frac{5}{\log 3}$$

B $x = \dfrac{50}{\log 3}$

INCORRECT: The student simplified log 10 as 10.

C $x = \dfrac{1}{5 \log 3}$

INCORRECT: The student divided the right-hand side of the equation by 5 instead of multiplying by 5.

D $x = \dfrac{\log 50}{\log 3}$

INCORRECT: The student simplified $5 \log 10$ as $\log 50$.

CCSS Conceptual Category:
Functions

CCSS Domain:
Linear, Quadratic, and Exponential Functions

5 **Find the vertical asymptote line(s) of the rational function $y = \frac{x^2 - 4x + 4}{x^2 - 4}$.**

A $x = 2$

INCORRECT: The student used the constant term in the expression left in the denominator.

B $x = -2$

CORRECT: Found the zero of the denominator:

$$y = \frac{x^2 - 4x + 4}{x^2 - 4}$$

$$y = \frac{(x - 2)(x - 2)}{(x - 2)(x + 2)}$$

$$y = \frac{x - 2}{x + 2}$$

$x + 2 = 0 \rightarrow x = -2$

[Note: there is a hole at $x = 2$.]

C $x = -2$ and $x = 2$

INCORRECT: The student did not simplify the common factor of $x - 2$.

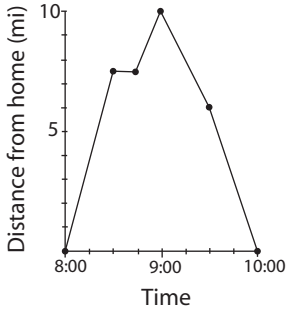
D $x = -4$ and $x = 4$

INCORRECT: The student factored the denominator incorrectly.

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions

6 **Zach went on a bike ride. He recorded his distance from home at various times along the route. Using the line graph that charts his progress, during which time period was Zach traveling at the greatest rate of speed?**



A From 8:00 to 8:30

CORRECT: The greatest rate of speed corresponds with the segment with greatest slope. The slopes of the segments corresponding to the answer options are $\frac{7.5}{30}, \frac{2.5}{15}, \frac{4}{30}, \frac{6}{30}$.

B From 8:45 to 9:00

INCORRECT: The student selected the time period that ends when Zach is the farthest from home.

C From 9:00 to 9:30

INCORRECT: The student selected the time period that begins when Zach is the farthest from home.

D From 9:30 to 10:00

INCORRECT: The student selected the time period with the greatest negative slope.

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions

7 **U-Rent charges $20 per day and 10 cents per mile to rent a car. Atlas charges $30 per day and 5 cents a mile. If you were renting a car for one day, when would U-Rent be cheaper than Atlas?**

A When driving less than 200 miles

CORRECT: The total cost to rent from U-Rent is represented by the expression $20 + 0.10x$, where x is the number of miles the car is driven. The total cost in dollars to rent from Atlas is represented by the expression $30 + 0.05x$, where x is the number of miles the car is driven:

$$20 + 0.10x < 30 + 0.05x$$
$$0.05x < 10$$
$$x < 200$$

B When driving more than 200 miles

INCORRECT: The student found when it would be more expensive to rent from U-Rent than Atlas.

C U-Rent is always cheaper than Atlas.

INCORRECT: The student answered based on the daily rate.

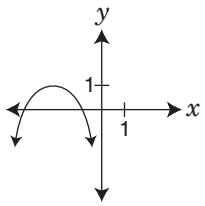
D U-Rent is never cheaper than Atlas.

INCORRECT: The student answered based on the mileage rate.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities

8 **What is the domain of the quadratic function graphed below?**



A $(-\infty, 0)$

INCORRECT: The student thought the graph shows a vertical asymptote at 0.

B $(-\infty, 1]$

INCORRECT: The student found the range.

C $(-\infty, \infty)$

CORRECT: All unrestricted quadratic functions have domains of all real numbers.

D $[1, \infty)$

INCORRECT: The student selected the part of the y-axis from the maximum point and greater.

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions

9 **If the discriminant of a quadratic equation is −25, what must be true about the roots of the equation?**

A They are imaginary conjugates.

CORRECT: A discriminant that is less than 0 indicates imaginary conjugate roots.

B They are irrational.

INCORRECT: The student confused imaginary with irrational.

C They are rational.

INCORRECT: The student thought that since the absolute value of the discriminate is a perfect square, the roots are rational.

D The equation has no roots.

INCORRECT: The student did not open the possibility of having roots outside of the real number system.

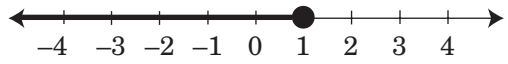
CCSS Conceptual Category:
Number and Quantity Overview

CCSS Domain:
The Complex Number System

10 **Graph the solution to the following inequality.**

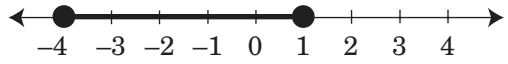
$$|2x + 3| \leq 5$$

A



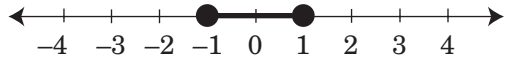
INCORRECT: The student only solved $2x + 3 \leq 5$ since the 5 was positive.

B



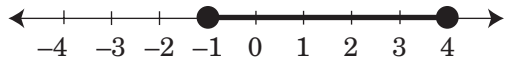
CORRECT:
$|2x + 3| \leq 5$
$-5 \leq 2x + 3$ and $2x + 3 \leq 5$
$-8 \leq 2x$ $2x \leq 2$
$-4 \leq x$ $x \leq 1$
$-4 \leq x \leq 1$
The graph of which shows closed dots at –4 and 1 and everything between the dots is shaded.

C



INCORRECT: The student added 3 and –5 in the left-hand side of the inequality.

D



INCORRECT: The student added 3 and –5 in the left-hand side and added 3 and 5 in the right-hand side of the inequality.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities

11 **What are the solutions to the following equation?**

$$2x^2 + 4x = -10$$

A $x = \pm 2i$

INCORRECT: The student did not take the opposite of b in the quadratic formula and then when simplifying the fraction thought the 2s divided to 0.

B $x = -1 \pm \sqrt{6}$

INCORRECT: The student moved the −10 to the left side of the equation without making it its opposite and solved: $2x^2 + 4x - 10 = 0$.

C $x = -1 \pm 2i$

CORRECT:

$2x^2 + 4x = -10 \rightarrow 2x^2 + 4x + 10 = 0 \rightarrow$
$2(x^2 + 2x + 5) = 0 \rightarrow x^2 + 2x + 5 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$$x = -1 \pm 2i$$

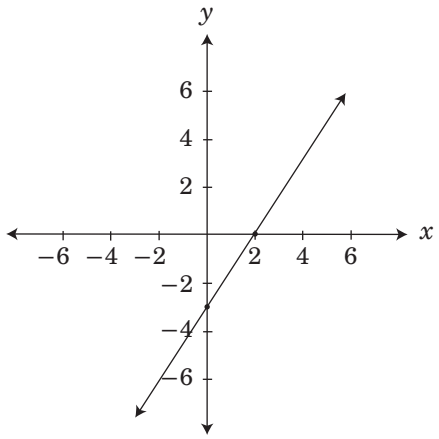
D $x = -1 \pm 4i$

INCORRECT: The student did not divide the second term in the numerator by 2.

CCSS Conceptual Category:
Number and Quantity Overview

CCSS Domain:
The Complex Number System

12 **What is the equation of the line that is perpendicular to and has the same *y*-intercept as the line graphed below?**



A $-2x + 3y = -9$

INCORRECT: The student used the reciprocal of the slope of the given line for the slope of the new line.

B $-3x + 2y = -6$

INCORRECT: The student used the slope of the given line which resulted in the equation of the given line.

C $2x + 3y = -9$

CORRECT: The slope of the given line is $\frac{3}{2}$, so the slope of a perpendicular line would be $-\frac{2}{3}$, (opposite reciprocal). The *y*-intercept of the given line is -3. Using slope-intercept form:

$$y = -\frac{2}{3}x - 3$$

$$3y = -2x - 9$$

$$2x + 3y = -9$$

D $3x + 2y = -6$

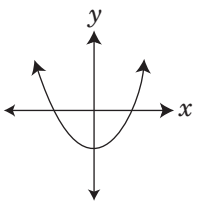
INCORRECT: The student used the opposite of the slope of the given line for the slope of the new line.

CCSS Conceptual Category:
Algebra

CCSS Domain:
Reasoning with Equations and Inequalities

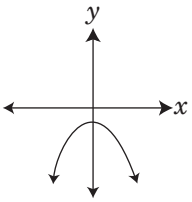
13 **Which of the following graphs represents a function that has no real roots?**

A



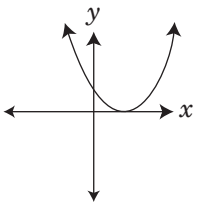
INCORRECT: The student selected a graph that has 2 real roots, one positive and one negative.

B



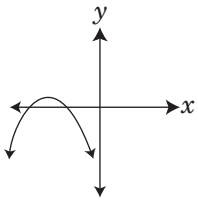
CORRECT: Since the graph of the function does not intersect the x-axis, the roots of the function are imaginary conjugates and therefore are not real.

C



INCORRECT: The student selected a graph that intersects the x-axis in one place and therefore has a double real root.

D



INCORRECT: The student selected a graph that has 2 real roots that are both negative.

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions

14 **What is the inverse of $y = \log(x - 3) + 4$?**

A $y = 10^{x-3} - 4$

INCORRECT: The student raised the log value to the power of 10 and took the opposite of the constant.

B $y = 10^{x-3} + 4$

INCORRECT: The student raised the log value to the power of 10.

C $y = 10^{x-4} + 3$

CORRECT: To find the inverse, switch the variables in the equation and solve for y.

$$x = \log(y - 3) + 4$$
$$x - 4 = \log(y - 3)$$
$$y - 3 = 10^{x-4}$$
$$y = 10^{x-4} + 3$$

D $y = 10^{x+4} + 3$

INCORRECT: The student added 4 to the left-hand side of the equation instead of subtracting 4.

CCSS Conceptual Category:
Functions

CCSS Domain:
Linear, Quadratic, and Exponential Functions

15 **What is true about the rate of change of the function $y = x^2$?**

A It decreases as x increases.

INCORRECT: The student was thinking of the value of the function as x approaches 0 from the left.

B It increases as x increases.

CORRECT: As x increases, the slopes of the tangent lines at each point are increasing from a limit of negative infinity to a limit of positive infinity.

C It decreases as $|x|$ increases.

INCORRECT: The student misinterpreted the steepness of the tangent lines as the function approaches negative infinity from the right and positive infinity from the left.

D It increases as $|x|$ increases.

INCORRECT: The student thought only about how steep the tangent lines would be approaching negative infinity from the right and positive infinity from the left and not whether the slopes are positive of negative.

CCSS Conceptual Category:
Functions

CCSS Domain:
Interpreting Functions